a number of related subjects should much more than make up for the time apparently lost in repetitions.

It seems, then, that we have three things that we should aim to do: first, we should aim to develop the student's technique in using with some freedom as many as possible of the great algorisms we naturally associate with the calculus; second, we should aim to present as far as possible problems which the student will recognize as applications of mathematics; and third, we should aim to give as broad an outlook as possible on the classical problems of analysis.

Granting that these should be our aims, there arise a number of questions as to methods of procedure and questions as to what should be the actual topics taken up. First of all, it is obviously undesirable to try to build up our calculus from a few simple postulates in a manner satisfactory from the point of view of formal logic, for time would not permit, even if we assumed that the student could appreciate such a method. On the contrary, it is desirable to make a great number of assumptions, subject of course to the condition that they be consistent, and further that the student will agree to their validity intuitionally.[1] The more we reflect on the fundamental postulates and elementary theorems involved in questions in limits and continuity, the more important it appears that the specialist in mathematics should eventually go into these questions deeply, but also the more important it seems that in the earlier stages of his training he shall make a free use of his geometric intuitions in such questions under the guidance of a more sophisticated intuition. I would therefore urge that we use our geometric intuitions with some freedom in this course, the instructor holding himself responsible for their correctness.

Furthermore is it wise to insist on giving proofs of all, or even of approximately all, of the theorems we may wish to state, even when those theorems do not appear intuitional to the student? The fact that mathematics is built up as a matter of logical proof from a set of axioms gives the subject a special claim for our attention, but is it advisable to require our students to go through the proofs of all, or of approximately all, of the theorems they may ever want to use?

My belief is that we would do better if we took greater freedom in stating theorems without proof, and spent more time in giving our students an idea of the topic under discussion in its entirety, incidentally requiring them to build higher through a more frequent acceptance of the dicta of others. This involves no necessary loss in the training of the student's reasoning powers, for the proofs that are given need be given no less carefully. The student may feel the lack of the completeness which is one of the charms of mathematics, and this would be serious if it came too early in his course, but it seems to me that the gains far exceed the losses if this policy is adopted in the course we are discussing.

As to the standard of rigor to be maintained in the proofs which are given, it is clear, with our expressed aims, that there should not be any radical change from the standards with which the student is already familiar. It is more

[1] See E. H. Moore: "On the Foundations of Mathematics," *Bulletin of the American Mathematical Society*, Vol. 9, 1902–1903, for remarks on the use of our intuition in elementary mathematics.

important to give arguments in a form which will be readily understood by the student and which are essentially complete and exact, even if not perfectly satisfactory from the point of view of formal logic, than to lay more stress on the logical minutiæ of the proofs. Geometrical intuitional arguments which are essentially conclusive should be used with freedom whenever they will aid in clearness of presentation. But arguments which involve actual misstatements, or which the best students may easily see to be illogical or fundamentally incomplete, should of course be avoided. Furthermore, carelessness in such things as dividing by zero, neglecting one of the square roots of a quantity, or assuming the converse of a proposition should not be permitted. The last error particularly is to be avoided, for this is more than a mathematical error—it is an element of daily reasoning which it is perhaps as important for the student to appreciate as any of the mathematical theory which he will learn.

Another thing which should perhaps be mentioned is the importance to be attached to problems in the course, and the character of the problems. Professor Osgood has said:[1] "The process by which the youth actually acquires the ideas of the calculus is to a large extent and essentially through formal work of substantial character. In order to attain this end however the formal work must appear to him as having for its direct object the power to solve some of the real problems of pure and applied mathematics, and those problems must be kept before his eye." My own experience as student and teacher bears out these statements. In the presentation of the course the solution of problems should constitute a vital part, a superficially more important part than the development of the theory. In this way the student's technique is developed, he obtains an appreciation of the utility of the subject, and he may be led to a consideration of many of the great old problems.

It is my purpose to take up in a moment an explicit list of topics suitable for the course under consideration, chosen in an attempt to meet the aims we have stated. Owing to differences in the first course as given at different times and places it is found necessary to include some topics which are often adequately treated in the first course in the calculus. One would naturally wish to avoid repetition except in the case of the most fundamental notions, where a review is worth while, but these topics are included so that they will not be overlooked in both courses in calculus. Recalling that we assume as a prerequisite only a three-hour year course in calculus, we see why there will be a considerable overlapping of topics with those often considered in a first course or in other courses of about the same grade.

The topics will be taken up in the order in which they might well be given to the class, dealing with partially known and simpler topics first, and leading to new and rather more difficult topics last,—following this out as far as seems compatible with a natural desire to associate closely related topics.

As an introductory chapter we need a brief review of the ideas of function,

[1] In his presidential address to the American Mathematical Society, "The Calculus in Our Colleges and Technical Schools," April 27, 1907; see the *Bulletin of the American Mathematical Society*, Vol. 13, 1906–1907, pp. 449–467.

limit, continuity, derivative, and integral. It is more important here to call attention again to the fundamental importance of the idea of function in applications of mathematics than to attempt any refinements in the definitions of limits and continuity. It is desirable to include a classification of the elementary functions, a discussion of their discontinuities, and a working test for locating discontinuities of a function of a function.[1] By introducing hyperbolic functions here we can both make it possible to use these important functions later and at the same time provide some new material for practice in the old methods of differentiation and integration.

A chapter on the simplest types of differential equations of the first and second orders can be given as a simple application of differentiation and integration. A discussion of families of curves, orthogonal trajectories, and the solution of numerous problems in geometry and mechanics will show the utility of the subject to the student. The importance and significance of the constants of integration should receive emphasis as well as the formal solution of equations.

This may be followed by a chapter leading up to and including Taylor's Theorem with Remainder. Starting from Rolle's Theorem and the simpler Law of the Mean as geometrically evident, we may prove the extension of the latter theorem to the quotient of two functions, and establish l'Hospital's Rule for the evaluation of indeterminate forms.[2] These theorems may be applied to the geometrical problem of contact of plane curves and to the problem of finding polynomial approximations to functions. This leads directly to Taylor's Theorem with Remainder, which is illustrated with special functions and is applied in computations and applications to extremes and points of inflection.

The consideration of R_n, the Remainder Term in Taylor's Theorem, as a function of n leads to infinite series. Convergence tests, including the comparison test, Cauchy's integral test and d'Alembert's ratio test, the simplest theorems on alternating series and absolute convergence, and a statement without proof of theorems concerning power series and uniform convergence, constitute the theory of series. They may be applied to various computations and to the solution of differential equations.

The consideration of a definite integral as the limit of a sum is closely related to infinite series, and though the subject is treated in the first course in calculus, its fundamental importance makes it merit further emphasis. Taking this up now, we may include approximation formulas, evaluations by ingenious devices and by infinite series as well as by the simple use of the indefinite integral method. Comparison theorems and mean value theorems for definite integrals may be given and applied to the approximate evaluation of difficult integrals as well as to certain theoretical questions.

Improper integrals, with tests for convergence, follow the preceding. Im-

[1] The last point is one quite often overlooked and has in my experience caused many good students perplexity, finding as they do the assumption of continuity in nearly every theorem, without being able to tell whether the functions with which they deal are continuous.

[2] It is very questionable if the proof of the latter theorem for the form ∞/∞ is worth giving, considering its difficulty for the student.

portant special integrals, including the Eulerian integrals, provide applications for this topic.

A brief treatment of elliptic integrals and functions may now be introduced. Only the simplest properties and uses of these functions can be given, time not permitting a treatment of the general methods of reducing elliptic integrals to standard types, computation of the standard integrals, or a systematic development of the theory of elliptic functions.

Thus far we have considered only functions of a single variable. To introduce functions of two or more variables I find it necessary to devote some time first to solid analytics. Then partial derivatives, the total differential, change of variable, differentiation of implicit functions, Jacobians and Taylor's Series may be treated. Applications are made to small errors and to geometry of space, including tangent planes and lines, osculating planes, maxima and minima, curves on a sphere, cylinder and cone. A part of the preceding will be a review of what has been studied in the first course in calculus. Also the next topic, multiple integrals, will be partly a review. Here we may include the definition as a limit of a sum, a restatement, if necessary, of Duhamel's Theorem (or a substitute), and a variety of applications to problems in areas, volumes, mass, centroids, pressure, moments of inertia, and attraction. Line and surface integrals, Green's and Stokes's Theorem and applications form an important and interesting extension of the study of integrals, and naturally follow the preceding.

There remain a number of topics one would wish to discuss, but time will not suffice to present all of them, and it is not quite clear which should have preference. Of these, probability, calculus of variations, Fourier's Series, and a further study of differential equations could be given in the spirit of the preceding, and all have a strong claim for recognition on account of their importance in various realms of applied mathematics. Vector analysis and functions of a complex variable are less closely related to what has preceded, but are perhaps of even more fundamental importance to the physicist and engineer. On the other hand, the pure mathematician may feel a strong need for a thorough discussion of certain existence theorems or of transformation of infinite series, or of questions of double limits that arise in the calculus; but my belief is, in keeping with general aims we have stated, that these topics must be sacrificed. It is quite obvious that there will be a lack of time for an extensive treatment of any of these topics, and it would doubtless be best to attempt to discuss only two or three of them. As to which these should be, I have at present no decided opinion. Perhaps we should, on general principles, allow for that much latitude in difference of opinion, and not even try to particularize further.

A SYSTEM OF ALGEBRAIC AND TRANSCENDENTAL EQUATIONS.[1]

By G. N. BAUER and H. L. SLOBIN, University of Minnesota.

The equations under consideration are of the form

$$P_1(x, y) = 0, \qquad P_2(x, y) = 0, \qquad T(x, y) = 0, \tag{1}$$

where $P_1(x, y)$ and $P_2(x, y)$ are polynomials in x and y with algebraic coefficients, and where $T(x, y) = 0$ denotes an equation which, in general, is not satisfied if both x and y are algebraic numbers. The equation $T(x, y) = 0$ may be satisfied by a finite number of pairs of algebraic numbers. Thus, the equation $x - e^y = 0$ is satisfied for $x = 1$, $y = 0$ and for no other pair of algebraic numbers. Other equations of this type are

$$P_1(x, y) + P_3(x, y) \tan P_2(x, y) = 0, \qquad P_1(x, y)e^{P_2(x, y)} + P_3(x, y) = 0,$$

where $P_1(x, y)$ and $P_3(x, y)$ have no common factor.

It is readily seen that there exists a non-enumerable set of systems of the type (1), for $T(x, y) = 0$ may be of the form $T(x, y) \equiv P_1(x, y) + t = 0$ where t is a transcendental number. This equation is not satisfied for a single pair of algebraic numbers.[2]

Throughout this paper $P_i(x, y)$ represents a polynomial in x and y with algebraic coefficients. The curve represented by an equation of the form $P_i(x, y) = 0$ will be called a P curve. The letter t always represents a transcendental number, and $p_i(t)$ represents a polynomial in t with algebraic coefficients. The curves represented by equations of the form $T(x, y) = 0$ are called T curves.

Before considering the system (1) attention is directed to a few preliminary considerations.

THEOREM 1. *The elimination of x or y from the equations*

$$P_1(x, y) = 0, \qquad P_2(x, y) = 0,$$

produces a polynomial equation in one unknown, with algebraic coefficients.

The elimination of either x or y gives a polynomial equation whose coefficients are derived from the coefficients of the two given polynomials by the operations of addition and multiplication. Hence the theorem is evident.[3, 4]

THEOREM 2. *The equations*

$$P_1(x, y) = 0, \qquad P_2(x, y) = 0,$$

are satisfied simultaneously by algebraic numbers only.

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[1] Read before the Chicago Section of the American Mathematical Society, December 22, 1916. *Bull. Am. Math. Soc.*, Vol. 23, p. 256.

[2] Bauer, G. N., and Slobin, H. L., "Some Transcendental Curves and Numbers," *Rendiconti del Circolo Matematico di Palermo*, Vol. 36, 1916, pp. 327–332.

[3] Burnside and Panton, *Theory of Equations*, p. 349.

[4] Bachmann, *Vorlesungen über die Natur der Zahlen*, 1892, p. 20.

This follows directly from theorem 1, for upon eliminating y, for example, the resulting equation can be satisfied by algebraic numbers only.[1]

THEOREM 3. *The system*

$$P_1(x, y) = 0, \qquad P_2(x, y) = 0, \qquad T(x, y) = 0,$$

can have no simultaneous solution except possibly one or more of the pairs of algebraic numbers which satisfy $T(x, y) = 0$.

This is apparent since, by theorem 2, the equations $P_1(x, y) = 0$, $P_2(x, y) = 0$ can be satisfied only by pairs of algebraic numbers, while, in general, $T(x, y) = 0$ cannot be satisfied by such a pair of numbers.

Thus, the system

$$P_1(x, y) = 0, \qquad P_2(x, y) = 0, \qquad y - \sin x = 0,$$

can have no simultaneous solution excepting possibly (0, 0).

Likewise, the system

$$P_1(x, y) = 0, \qquad P_2(x, y) = 0, \qquad P_3(x, y) - t = 0,$$

can have no simultaneous solution.

It is evident that no two P curves can intersect on a T curve unless, by way of exception, the isolated points on the curve represented by $T(x, y) = 0$ whose coördinates are both algebraic numbers, happen to be among the intersections of the two P curves.

If a P curve cut a T curve, no other P curve can pass through any transcendental point of intersection; hence any curve which passes through a transcendental point of intersection of a P curve and a T curve is not a P curve. It does not follow from this that the curve is a T curve since it has not been shown that the P and T curves exhaust all possibilities. In fact, the equation

$$P_1(x, y)P_2(x, y) + tP_1(x, y) = 0$$

represents a curve that is neither a P curve nor a T curve.

THEOREM 4. *The equation* $P_1(x, y) = 0$ *can be satisfied only by a pair of numbers both of which are algebraic or both of which are transcendental.*

Let x be an algebraic number, then $P_1(x, y) = 0$ may be considered a polynomial with algebraic coefficients; hence y must be an algebraic number. If on the other hand x is a transcendental number, y cannot be algebraic, otherwise a transcendental number would satisfy an algebraic equation with algebraic coefficients.

THEOREM 5. *If a P curve (not a straight line) intersect a T curve (not in its isolated algebraic points), then the slope of any line through any algebraic point of the plane and through any point of intersection is a transcendental number.* (Any point of the plane whose coördinates are algebraic numbers is called an algebraic point.)

[1] Bachmann, *ibid.*, p. 21.

Let A be any intersection of a P curve and a T curve. Its coördinates are both transcendental numbers (t_1, t_2) since by hypothesis they cannot both be algebraic, and by theorem 3 one cannot be algebraic while the other is transcendental. Let (a_1, a_2) be the coördinates of any algebraic point of the plane. Let us assume that the slope is algebraic; then the equation is of the form

$$y - a_2 = m(x - a_1),$$

where m is an algebraic number. Then we have three equations, two representing P curves and one representing a T curve, intersecting in a common point, both of whose coördinates are transcendental numbers. But this is impossible by theorem 3. Hence m must be a transcendental number.

If the P curve is a straight line the theorem is true for all algebraic points of the plane not on the line. The theorem obviously does not apply to the algebraic points on the line, if P be a line, since the equation of the P curve is an algebraic equation, and hence the slope is an algebraic number.

Also, if the given P curve is a line, and if the slope in question is an algebraic number, then any algebraic point through which the line may be drawn must lie on the given curve, *i. e.*, the line so constructed must coincide with the given P line.

Corollary 1. Any line with an algebraic slope, passing through the transcendental intersections of a P and a T curve, does not pass through any algebraic point.

If the line pass through an algebraic point, it would be possible to write the equation in terms of the coördinates of the point and the slope, and hence the line would be a P curve. Then two P curves and a T curve would intersect in a transcendental point, which is impossible.

Corollary 2. If the equation of a P curve is of the form

$$P_1(x, y) - kP_2(x, y) = 0$$

where k is any algebraic number, the multiple points of the P curve due to the intersection of $P_1(x, y) = 0$ and $P_2(x, y) = 0$ cannot be among the transcendental points on a T curve.

THEOREM 6. *If a P curve (not a circle) cut a T curve (not in its algebraic points), then the distance between a point of intersection and any algebraic point of the plane is a transcendental number.*

Let the coördinates of any algebraic point of the plane be given by (a_1, a_2), and let d be the distance from this point to a point of intersection of a P curve and a T curve. Let us assume that d is an algebraic number. Then, with (a_1, a_2) as a center pass a circle through the point of intersection. The radius of the circle is d. Also the equation of the circle is given in terms of a_1, a_2 and d, and hence it is a P curve. We then have two P curves intersecting a T curve in a transcendental point which, by theorem 3, is impossible. Hence d is not an algebraic number, and must therefore be transcendental.

If the given P curve is a circle, the theorem is still true for all algebraic points of the plane with the exception of the center of the given circle. The theorem evidently does not apply to the center of the circle.

Corollary. If a circle which is a P curve cut a T curve (not in its isolated algebraic points), then the center of the circle is the only algebraic point in the plane whose distance from a point of intersection is an algebraic number.

THEOREM 7. *No circle with an algebraic radius, r, whose center is an intersection of a P curve and a T curve, passes through an algebraic point of the plane* (where r is not equal to the radius of the P curve, in case P happens to be a circle).

For, if the circle passes through an algebraic point (a_1, a_2), it is possible to construct a circle with this point as a center, and r as a radius. It would therefore be a P curve, and two P curves would intersect a T curve, which is impossible.

THEOREM 8. *If a tangent or a normal to a P curve passes through an algebraic point* (a_1, a_2), *it cannot pass through the intersection of the P curve and a T curve, unless the intersection happens to be one of the finite number of isolated algebraic points of the T curve.*

The equations of the tangents and normals to the P curve, passing through the point (a_1, a_2) can be expressed in terms of a_1, a_2 and the partial derivatives of $P(x, y)$. Hence these equations represent P curves. But it is impossible for two P curves and a T curve to intersect; hence the theorem.

Corollary. A line tangent to a P curve at an algebraic point is a P curve.

THEOREM 9. *Any normal* [*tangent*] *line to a P curve, through any algebraic point* (a_1, a_2) *of the plane is normal* [*tangent*] *at an algebraic point.*

This follows directly from the fact that under the stated conditions, the tangent and normal are P curves.

THEOREM 10. *Any line drawn tangent* [*normal*] *to a P curve at a transcendental point is a T curve.*

For, if it contained an algebraic point, we could set up

$$a_1 \frac{\partial P(x, y)}{\partial x} + a_2 \frac{\partial P(x, y)}{\partial y} = 0, \tag{1}$$

$$(a_1 - x) \frac{\partial P(x, y)}{\partial y} - (a_2 - y) \frac{\partial P(x, y)}{\partial x} = 0, \tag{2}$$

respectively the curve passing through the points of contact of the tangents drawn from (a_1, a_2) to $P(x, y) = 0$, and the curve passing through the feet of all normals drawn from (a_1, a_2) to $P(x, y) = 0$, and find two P curves intersecting in a transcendental point. But this is impossible. Hence it is a T curve.

If the P curve is a straight line, the first member of (1) is *constant*, and the theorem does not apply. If the P curve is a circle and (a_1, a_2) the center of circle (2) is identically 0, and the theorem does not apply in that case.

THEOREM 11. *No transcendental point of a P curve is a singular point.*

For the singular points demand the simultaneous existence of

$$P(x, y) = 0, \qquad \frac{\partial P(x, y)}{\partial x} = 0, \qquad \frac{\partial P(x, y)}{\partial y} = 0,$$

which represent P curves and can be satisfied simultaneously only by algebraic

numbers. If one of the derivatives reduces identically to 0, the remaining equation must be satisfied simultaneously with the given equation $P(x, y) = 0$. This demands that the solution be algebraic numbers.

Corollary. A T curve cannot intersect a P curve in the singular points of the P curve, unless the singular point happens to be one of a finite number of algebraic points of the T curve.

THEOREM 12. *All of the singular points of*

$$R(x, y) \equiv P_1(x, y) + p(t)P_2(x, y) = 0, \qquad [1]$$

which require

$$\frac{\partial R(x, y)}{\partial x} = 0 \qquad [2]$$

and

$$\frac{\partial R(x, y)}{\partial y} = 0, \qquad [3]$$

are among the intersections of $P_1(x, y) = 0$ and $P_2(x, y) = 0$, if there are any at all.

The equations [2] and [3], in general, contain t. Eliminating t between [1] and [2] and also between [2] and [3] we have two equations representing P curves, which, taken together with [1] give a system which cannot be satisfied simultaneously excepting for the values common to $P_1(x, y) = 0$ and $P_2(x, y) = 0$.

Corollary. The curves represented by the equations of the type $P_1(x, y) + p(t) = 0$ have no singular points.

THEOREM 13. *All the singularities of*

$$R(x, y) \equiv p_1(t)P_1(x, y) + p_2(t)P_2(x, y) = 0$$

are found among the intersections of $P_1(x, y) = 0$ and $P_2(x, y) = 0$, if any exist, provided $p_1(t) \not\equiv \lambda p_2(t)$.

By setting up the partial derivatives of $R(x, y)$ with respect to x and y, the argument is seen to follow the lines used in the last demonstration.

THEOREM 14. *The singularities of*

[1] $$R(x, y) \equiv p_0(t)P_0(x, y) + p_1(t)P_1(x, y) + \cdots + p_n(t)P_n(x, y) = 0,$$

if any exist, are among the simultaneous solutions of

$$P_0(x, y) = 0, \qquad P_1(x, y) = 0, \qquad \cdots, \qquad P_n(x, y) = 0$$

provided

[2] $$\| a_0, b_1, c_2, \cdots, k_n \| \neq 0$$

where the elements entering the matrix are the coefficients of the polynomials $p_0(t)$, $p_1(t)$, $\cdots$, $p_n(t)$.

Let us write

$$p_i(t) = a_i + b_i t + c_i t^2 + \cdots \quad [i = 0, 1, 2, \cdots n]$$

and let the highest power of t occurring in any polynomial be designated by j.

Also, representing $P_i(x, y)$ by A_i it is seen that the following $j + 1$ equations must subsist in order that there may be a solution:

$$\begin{aligned} a_0A_0 + a_1A_1 + \cdots + a_nA_n &= 0, \\ b_0A_0 + b_1A_1 + \cdots + b_nA_n &= 0, \\ \cdot \quad\quad \cdot \quad\quad\quad \cdot & \\ \cdot \quad\quad \cdot \quad\quad\quad \cdot & \\ \cdot \quad\quad \cdot \quad\quad\quad \cdot & \\ k_0A_0 + k_1A_1 + \cdots + k_nA_n &= 0; \end{aligned}$$

for the coefficients of the individual powers of t must reduce to 0, since by placing

$$\frac{\partial R(x, y)}{\partial x} = 0 \qquad \text{and} \qquad \frac{\partial R(x, y)}{\partial y} = 0$$

it is seen that the singular points must be algebraic points. This requires that the matrix [2] be equal to 0 or that $A_0 = A_1 = \cdots = A_n = 0$. But by hypothesis the matrix is not equal to 0 and hence $A_0, A_1, \cdots, A_n$ must reduce to 0, i. e., the singularities are among the simultaneous solutions of $P_0 = 0$, $P_1 = 0$, $\cdots$, $P_n = 0$.

Corollary. The total number of singular points, which require

$$R(x, y) = 0, \qquad \frac{\partial R(x, y)}{\partial x} = 0, \qquad \frac{\partial R(x, y)}{\partial y} = 0,$$

cannot exceed jk where j and k are the degrees of the two polynomials of lowest degree, which enter in [1].

It is evident that many such equations [1] may occur where jk would be much less than the maximum number of such points possible as determined from the table of Plückerian Characteristics. Thus for example

$$p_1(t)x + p_2(t)y + p_3(t)P_n(x, y) = 0$$

where $P_n(x, y)$ is a polynomial of the nth degree with the absolute constant not 0, has no such singularities, for $x = 0$, $y = 0$ do not satisfy $P_n(x, y) = 0$.

The various theorems pertaining to singularities may easily be extended to geometry of higher dimensions.

PRACTICAL SOLUTION OF LINEAR EQUATIONS.

By H. T. BURGESS, University of Wisconsin.

If we attempt to find a Fundamental System of Solutions[1] of a system of linear equations by the methods found in the literature,[2] we meet with considerable difficulty in the application.

It is the purpose of this brief paper to explain a scheme which works with remarkable ease and simplicity in practice, and which is likewise available for theoretical purposes, if we so desire to use it.

1. **Preliminary Matrix Theory.** If the rectangular matrix

$$A' = \begin{Vmatrix} a_{11} & a_{21} & \cdots & a_{m1} \\ a_{12} & a_{22} & \cdots & a_{m2} \\ \cdot & \cdot & \cdot & \cdot \\ a_{1n} & a_{2n} & \cdots & a_{mn} \end{Vmatrix}$$

is of rank r,[3] we can reduce A' by elementary transformations[4] on the rows only to the form $\begin{Vmatrix} R \\ Z \end{Vmatrix}$, where Z consists of $(n - r)$ rows of zeros.

One method of procedure is as follows: (1) if $a_{11} = 0$, some $a_{1j} \neq 0$, interchange rows 1 and j; (2) divide row 1 by a_{11}; (3) multiply row 1 successively by $a_{12}, a_{13}, \cdots, a_{1n}$ and subtract the successive products from rows 2, 3, $\cdots$, n respectively; (4) continue this process until either (a) r columns have been so reduced or (b) some column, say the kth, $k < r$, is found where every element below the $(k - 1)$st row is zero. In case (a), the resulting matrix has the form $\begin{Vmatrix} I & S \\ Z_1 & Z_2 \end{Vmatrix}$, where I is the unit matrix[5] of order r, and $(n - r)$ rows of zeros appear in Z_1 and Z_2. This must be the case, for the presence of a non-zero element in Z_2 would increase the rank of A' by one. In case (b), pass on to the first column, say the pth, in which a non-zero element appears in or below the kth row. Interchange rows, if necessary, so that $a_{kp} \neq 0$. Divide the kth row by a_{kp} and reduce all other elements in the pth column to zero as in (3). Continue this process until r columns, exclusive of those passed over, are reduced as in (3). The result is in the form

$$\begin{Vmatrix} J & P \\ Z_1 & Z_2 \end{Vmatrix},$$

where J is of rank r and has r rows, and $(n - r)$ rows of zeros appear in Z_1 and Z_2. That the rank of J is r is apparent if we omit the columns which were passed over

[1] See Bôcher's *Introduction to Higher Algebra*, p. 50.
[2] See Bôcher, loc. cit.
[3] See Bôcher, pp. 20–22.
[4] See Bôcher, p. 55.
[5] See Bôcher, p. 74.

and observe that the remaining matrix is the unit matrix I of order r. As before, a non-zero element can not appear in Z_2, for this would increase the rank of A' by one.

2. **Solution of a System of Linear Homogeneous Equations.** The system

$$(a)\qquad \begin{aligned} a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n &= 0,\\ a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n &= 0,\\ \cdots\cdots\cdots\

We write

$$\left\|\begin{array}{rrrr|rrrrr} 1 & 1 & -2 & -7 & 1 & 0 & 0 & 0 & 0 \\ 11 & 2 & 7 & 4 & 0 & 1 & 0 & 0 & 0 \\ -2 & -1 & 1 & 5 & 0 & 0 & 1 & 0 & 0 \\ 8 & 3 & -1 & -11 & 0 & 0 & 0 & 1 & 0 \\ 3 & 2 & -3 & -12 & 0 & 0 & 0 & 0 & 1 \end{array}\right\|.$$

(1) Add rows 4 and 5 and subtract from row 2;
(2) Multiply row 3 by 4 and add to row 4;
(3) Add row 3 to row 5 and subtract row 1 from the result;
(4) Multiply row 1 by 2 and add to row 3;
(5) Add row 3 to row 4;
(6) Multiply row 3 by 3 and add to row 2;
(7) Interchange rows 2 and 3.

The result is

$$\left\|\begin{array}{rrrr|rrrrr} 1 & 1 & -2 & -7 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & -3 & -9 & 2 & 0 & 1 & 0 & 0 \\ 0 & 0 & 2 & 0 & 6 & 1 & 3 & -1 & -1 \\ 0 & 0 & 0 & 0 & 2 & 0 & 5 & 1 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & 1 & 0 & 1 \end{array}\right\|$$

and there are two solutions.

3. **Non-homogeneous Linear Equations.** If we set $x_n = 1$ in the system (a), we have a non-homogeneous system. The process of solution is the same except the last column of S must be reduced to a column of 1's by elementary transformations on the rows of S.

A necessary and sufficient condition for such a solution is that at least one element in the last column of S shall be different from zero. This is equivalent to the condition that the matrix and the augmented matrix shall have the same rank.[1]

Illustration. Solve the system

$$\begin{aligned} 4x - y + 5z + 1 &= 0, \\ 2x - 3y + z + 5 &= 0, \\ x + y + 2z - 2 &= 0, \\ 5x + 2z - 1 &= 0. \end{aligned}$$

We write

$$\left\|\begin{array}{rrrr|rrrr} 4 & 2 & 1 & 5 & 1 & 0 & 0 & 0 \\ -1 & -3 & 1 & 0 & 0 & 1 & 0 & 0 \\ 5 & 1 & 2 & 2 & 0 & 0 & 1 & 0 \\ 1 & 5 & -2 & -1 & 0 & 0 & 0 & 1 \end{array}\right\|.$$

[1] See Bôcher, pp. 44–46.

(1) Interchange rows 1 and 4;
(2) Multiply row 2 by 4 and add to row 4;
(3) Multiply row 2 by 5 and add to row 3;
(4) Add row 1 to row 2;
(5) Multiply row 2 by 5 and add to row 4;
(6) Multiply row 2 by 7 and add to row 3.

The result is

$$\left\|\begin{array}{cccc|cccc} 1 & 5 & -2 & -1 & 0 & 0 & 0 & 1 \\ 0 & 2 & -1 & -1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & -5 & 0 & 12 & 1 & 7 \\ 0 & 0 & 0 & 0 & 1 & 9 & 0 & 5 \end{array}\right\|.$$

and the solution is $x = \frac{1}{5}$, $y = \frac{9}{5}$, $z = 0$.

If the "5" in the last row and column had come out a zero, the equations would have been inconsistent.

PROBLEMS FOR SOLUTION.

SEND ALL COMMUNICATIONS ABOUT PROBLEMS TO B. F. FINKEL, Springfield, Missouri.

2732. Proposed by PAUL CAPRON, U. S. Naval Academy.

A conical cup, filled with a fluid, stands with the vertex on a smooth horizontal surface. The inner and outer surfaces of the cup are similar cones of revolution, having altitudes h and $h(1+x)$; the ratio of the specific weights of the material of the cone and the fluid is σ; the height of a barometer column of the fluid is h_0. Show that for equilibrium

$$h_0/h(1+x)^2 + \sigma x(1+x+x^2/3) \leqslant 2/3.$$

2733. Proposed by J. L. RILEY, Stephenville, Texas.

An ellipse of constant eccentricity passes through the focus of a parabola and has its foci on the curve. Find the envelopes of its axes.

2734. Proposed by E. L. REES, The University of Kentucky.

Given two circles tangent to each other externally. From the extremity of a diameter through the point of tangency, draw a secant such that the segment between the circles shall be equal to a given segment.

2735. Proposed by H. B. PHILLIPS, Massachusetts Institute of Technology.

If two lines AE and BD, drawn from the vertices, A and B, of a triangle to the opposite sides, divide the angles A and B so that the parts of A are respectively less than the corresponding parts of B, then AE is greater than BD.

2736. Proposed by M. COHEN, Freshman, Johns Hopkins University.

Prove by elementary geometry that the orthocenter, the centroid, and the circumcenter of a triangle lie on a line (the Euler line), and that the centroid lies between the other two and is twice as far from the orthocenter as from the circumcenter.

SOLUTIONS OF PROBLEMS.

2669. Proposed by S. A. COREY, Albia, Iowa.

Let $A_1, A_2, \cdots, A_8$, and $-(A_1 + A_2 + \cdots + A_8)$ be the vector sides of an enneagon, plane or gauche. Also let $B_1, B_2, \cdots, B_8$, and $-(B_1 + B_2 + \cdots + B_8)$ be the vector sides of a second enneagon, where $B_1 = C_1A_1 - C_2C_5A_2 - C_3C_6A_5 + C_4C_5C_6A_7$, $B_2 = C_1A_2 - C_2C_5A_4 - C_3C_6A_6 + C_4C_5C_6A_8$, $B_3 = C_2A_1 + C_1A_3 - C_4C_6A_5 - C_5C_6A_7$, $B_4 = C_2A_2 + C_1A_4 - C_4C_6A_6 - C_5C_6A_8$, $B_5 = C_3A_1 + C_4C_5A_2 + C_1A_5 + C_2C_5A_7$, $B_6 = C_3A_2 + C_4C_5A_4 + C_1A_6 + C_2C_5A_8$, $B_7 = C_4A_1 - C_3A_3 + C_2A_5 - C_1A_7$, and $B_8 = C_4A_2 - C_3A_4 + C_2A_6 - C_1A_8$, C_1, C_2, C_3, C_4, C_5, and C_6 being scalars.

Then, if a_r = tensor A_r, b_r = tensor B_r, and cos (A_rA_s) = cosine of the angle included between A_r and A_s and cos (B_rB_s) = cosine of the angle included between B_r and B_s, establish the following relation between the sides and angles of the two enneagons:

$$[C_1^2 + C_5C_2^2 + C_6C_3^2 + C_5C_6C_4^2][a_1a_2 \cos (A_1A_2) + C_5a_3a_4 \cos (A_3A_4) + C_6a_5a_6 \cos (A_5A_6) + C_5C_6a_7a_8 \cos (A_7A_8)]$$

$$= b_1b_2 \cos (B_1B_2) + C_5b_3b_4 \cos (B_3B_4) + C_6b_5b_6 \cos (B_5B_6) + C_5C_6b_7b_8 \cos (B_7B_8).$$

Show that Geometry problem 506 is a special case of the foregoing. Give illustrative example, using triangle or other simple geometric figure, by assuming that some of the sides of the first enneagon are zero.

SOLUTION BY PROPOSER.

Whenever $A_1, A_2, \cdots$ and A_8 are scalar (or algebraic) quantities, and $B_1, B_2, \cdots$ and B_8 have scalar values corresponding in form to those given in the problem, we have the algebraic identity,

$$(C_1^2 + C_5C_2^2 + C_6C_3^2 + C_5C_6C_4^2)(A_1A_2 + C_5A_3A_4 + C_6A_5A_6 + C_5C_6A_7A_8) = B_1B_2 + C_5B_3B_4 + C_6B_5B_6 + C_5C_6B_7B_8.$$

Inasmuch as all the terms in $A_1, A_2, \cdots A_8$, and $B_1, B_2, \cdots B_8$ in this algebraic identity are of the second degree, a geometric interpretation may be obtained by assuming that $A_1, A_2, \cdots A_8$ and $B_1, B_2, \cdots B_8$, are vectors. This follows immediately from the fact that vector multiplication is commutative in so far as the scalar part of the product is concerned whenever all the vector terms employed are of the second degree. But the scalar part of the vector product B_rB_s is $-b_rb_s \cos (B_rB_s)$. Substituting this scalar part of the vector product in both members of the above algebraic equation and changing signs we obtain at once the equation contained in the problem. If $A_1 = A_2$, $A_3 = A_4$, $A_5 = A_6$, and $A_7 = A_8$, the problem becomes identical with Geometry problem 506.

Example. As long as a vector maintains a constant length and direction in space, its origin in space may be altered at will. Hence we need not confine our attention to closed geometric figures in interpreting the given identity. Let DEF be a given triangle. Bisect DE in G. Draw GF and extend EF to F'. Draw GH intersecting DF in I and EF' in J. Let $A_1 = GF$, $A_2 = GI$, and $A_3 = GD$. Also let $A_4 = A_5 = A_6 = A_7 = A_8 = 0$, $C_1 = C_2 = C_3 = C_4 = C_5 = 1$, and $C_6 = 0$. Substituting in the given identity, paying strict attention to the direction of the vectors employed, and dividing by the constant factor, we readily get $2GF \cos FGI = DF \cos GID + EF \cos EJG$, a known result.

2670. Proposed by CLIFFORD N. MILLS, Brookings, South Dakota.

A telegraph wire, weighing one tenth pound per yard, is stretched between poles on level ground, so that the greatest dip of the wire is three feet. Find approximately the distance between the poles when the tension at the lowest point of the wire is 140 pounds.

SOLUTION BY ELBERT H. CLARKE, Hiram College, Ohio.

It is a well-known property of wires hanging freely from two supports that the tension at

any point is equal to that which would be produced by a wire of the same weight per unit length, hanging vertically and being of the same length as the ordinate of the point where the tension is measured, the equation of the curve being of the form,

$$y = \frac{a}{2}(e^{x/a} + e^{-x/a}).$$

Hence, the equation of the catenary curve assumed by the wire in the above problem will be

$$y = 700(e^{x/1400} + e^{-x/1400}),$$

the unit of distance being one yard.

The amount of dip or sag in a symmetrical segment of horizontal projection, $2h$, will be

$$d = \frac{a}{2}(e^{h/a} + e^{-h/a}) - a$$

$$= \frac{h^2}{2a} + \frac{h^4}{24a^3} + \frac{h^6}{720a^5} + \cdots.$$

In our problem $d = 1$, $a = 1400$, and a very close first approximation is given by neglecting all terms in the series after the first. Hence, $h = 52.915$, approximately. Using more terms the result correct to three places is $h = 52.912$. The distance between the poles, correct to the nearest tenth of a foot is 317.5 feet.

Also solved by ROGER A. JOHNSON.

2671. Proposed by ARTEMAS MARTIN, Washington, D. C.

Find two rectangular parallelopipedons whose edges are rational whole numbers and whose solid diagonals are also rational whole numbers and equal.

SOLUTION BY S. A. COREY, Albia, Iowa.

We have the identities

$$(x^2 + y^2 + u^2 + v^2)^2 = (x^2 - y^2 - u^2 + v^2)^2 + (2xy - 2uv)^2 + (2ux + 2vy)^2$$
$$= (x^2 - y^2 + u^2 - v^2)^2 + (2xy + 2uv)^2 + (2vx - 2uy)^2$$
$$= (x^2 + y^2 - u^2 - v^2)^2 + (2ux - 2vy)^2 + (2vx + 2uy)^2.$$

By letting x, y, u, and v represent rational whole numbers any number of solutions of the problem may be obtained. One such solution is obtained by letting $x = 1$, $y = 2$, $u = 5$, and $v = 7$, and we find *three* parallelopipedons fulfilling the requirements of the problem, with edges 21, 66, 38; 27, 74, 6; and 69, 18, 34, respectively, the solid diagonal of each being 79. The three smallest have edges, 1, 2, 2; 2, 4, 4; and 2, 3, 6, respectively.

Also solved by L. E. LUNN, W. P. DURFEE, and H. L. OLSON.

2673. Proposed by WILLIAM O. BEAL, University of Minnesota.

A plane through the center of an oblate spheroid makes an angle, i, with the plane of its equator. Express the eccentricity, e', of this section in terms of the eccentricity, e, of a meridian section and the angle, i.

SOLUTION BY C. A. BARNHART, Colorado College.

Let the equation of the oblate spheroid be $x^2/b^2 + y^2/b^2 + z^2/a^2 = 1$, $(b > a)$, of which the z-axis is the axis of revolution. Let $OP = b'$ be the semi-minor axis of the given plane section of which b is the semi-major axis. Then P will be a point on the meridian section, which may be assumed to lie in the yz-plane and to have the parametric equations:

$$z = a \sin \theta, \qquad y = b \cos \theta;$$

and the eccentricity,

$$e = \frac{\sqrt{b^2 - a^2}}{b}.$$

Since

$$e' = \frac{\sqrt{b^2 - b'^2}}{b}$$

and

$$b' = \sqrt{y^2 + z^2} = \sqrt{b^2 \cos^2\theta + a^2 \sin^2\theta},$$

$$e' = \frac{\sqrt{b^2 - b^2\cos^2\theta - a^2\sin^2\theta}}{b} = \frac{\sqrt{(b^2 - a^2)(1 - \cos^2\theta)}}{b} = e \sin\theta.$$

But $z/y = \tan i = (a/b)\tan\theta$, or $\tan\theta = (b/a)\tan i$. Therefore,

$$\sin\theta = \frac{b\tan i}{\sqrt{a^2 + b^2\tan^2 i}} = \frac{b\tan i}{\sqrt{b^2 - b^2e^2 + b^2\tan^2 i}} = \frac{\tan i}{\sqrt{\sec^2 i - e^2}}.$$

Substituting for $\sin\theta$, we have

$$e' = \frac{e\tan i}{\sqrt{\sec^2 i - e^2}}.$$

Also solved by ROGER A. JOHNSON, ELBERT H. CLARKE, H. L. OLSON, and the PROPOSER.

2674. Proposed by J. O. MAHONEY, Dallas, Texas.

If two sides of a triangle differ by less than a certain length, e, the two opposite angles will differ by less than a certain quantity λ, expressed in degrees, such that $\lambda < 61e/a$ where a expresses, with a possible error e, the length of the apparently equal sides of the triangle.

SOLUTION BY ROGER A. JOHNSON, Hamline University.

This theorem as stated, is not true. Consider, for instance, the triangle

$$a = 1001, \qquad b = 1000, \qquad c = 99.$$

Here,

$$A = 87^\circ\,44'\,32'', \qquad B = 86^\circ\,35'\,10'', \qquad C = 5^\circ\,40'\,18''.$$

Now, considering the nearly equal angles A and B, we have, in fact, $\lambda = 1.157$, whereas by the formula given, we should have $\lambda = .061$ or less. This is an extreme case, but it will be found that in any triangle in which the nearly equal angles are greater than about 50°, the formula does not hold.

As a matter of fact, the correct expression is

$$\lambda = \frac{180}{\pi}\frac{e}{a}\tan A,$$

where A represents the larger of the nearly equal angles. We will not consider the case that either of these two angles exceeds or equals 90°.

If a and b are two sides of a triangle, α, β, the opposite angles, we have

$$\sin\beta = \frac{\sin\alpha}{a}\cdot b;$$

whence

$$\cos\beta\, d\beta = \frac{\sin\alpha}{a}\, db,$$

in radians, or

$$d\beta = \frac{180^\circ}{\pi}\cdot\frac{\sin\alpha}{\cos\beta}\frac{db}{a}$$

For our problem, $d\beta = \lambda$, $db = e$, and $(\sin\alpha)/(\cos\beta)$ may be replaced by the greater of the values $\tan\alpha$, $\tan\beta$, yielding the inequality given above.

Also solved by ELIJAH SWIFT and PAUL CAPRON.

2675. Proposed by E. B. ESCOTT, Kansas City, Mo.

Sum the series

$$-\frac{1}{2}\cdot\frac{1^3x^3}{3}+\frac{1\cdot3}{2\cdot4}\cdot\frac{2^3x^5}{5}-\frac{1\cdot3\cdot5}{2\cdot4\cdot6}\frac{3^3x^7}{7}+\cdots.$$

SOLUTION BY ELIJAH SWIFT, University of Vermont.

Differentiate the given series (permissible, as a power series, convergent $|x|<1$). Dividing by x^2, we have the series,

$$P(x)=-\frac{1}{2}\cdot1^3+\frac{1\cdot3}{2\cdot4}2^3(x^2)-\frac{1\cdot3\cdot5}{2\cdot4\cdot6}3^3(x^2)^2+\cdots.$$

If we substitute z for x^2 and then integrate this from 0 to z, divide by z and integrate again, and repeat this process, we arrive at a familiar series. (We are integrating a function which is obviously continuous at $z=0$.) The resulting series is

$$-\frac{1}{2}z+\frac{1\cdot3}{2\cdot4}z^2-\frac{1\cdot3\cdot5}{2\cdot4\cdot6}z^3+\cdots=(1+z)^{-1/2}-1.$$

Reversing the processes we have carried out on the series, we finally obtain the original series in closed form. Thus, differentiating $(1+z)^{-1/2}-1$ and multiplying by z, repeating this process, and differentiating the result, we have, putting $z=x^2$,

$$P(x)=\frac{-x^4+10x^2-4}{8(1+x^2)^{7/2}}.$$

Multiplying this by x^2, integrating and determining the constant of integration, the value of the given series is

$$\frac{1}{8}\left\{\frac{3x}{(1+x^2)^{5/2}}-\frac{5x}{(1+x^2)^{3/2}}+\frac{3x}{(1+x^2)^{1/2}}-\log\left(x+\sqrt{1+x^2}\right)\right\}$$
$$=\frac{x+x^3+3x^5}{8\sqrt{(1+x^2)^5}}-\tfrac{1}{8}\log\left(x+\sqrt{1+x^2}\right).$$

Also solved by H. L. OLSON and the PROPOSER.

UNDERGRADUATE MATHEMATICS CLUBS.

EDITED BY R. C. ARCHIBALD, Brown University, Providence, R. I.

CLUB ACTIVITIES.

THE JUNIOR MATHEMATICAL CLUB OF THE UNIVERSITY OF CHICAGO, Chicago, Ill. [*1918*, 34–5].[1]

November 15, 1916: "Newton" by Gail F. Moulton '19 and Thomas McN. Simpson Gr.

November 29: "Elementary notions of line complexes and congruences" by Levi S. Shively Gr.

December 13: "Notes in the history of the theory of functions of a complex variable" by Joseph L. Walsh Gr.

January 14, 1917: "Quadratic forms" by James E. McAtee Gr.

January 28: "Transformation of Coördinates" by Kenneth Lamson Gr.

[1] This abbreviation indicates that on pages 34–35 of this MONTHLY, 1918, there may be found further information concerning The Junior Mathematical Club.

February 14: "An Application of Bayes's theorem[1] in the theory of probabilities" by Professor William D. Cairns of Oberlin College.
February 28: "Cauchy" by George H. Cresse Gr.; "Poncelet" by Ernest P. Lane Gr.; "Gauss" by Mary C. Suffa Gr.; "Weierstrass" by Webster G. Simon Gr.
April 11: "Elementary notions of continuous groups" by Israel A. Barnett Gr.
April 25: "Note on a set of postulates" by William P. Ott Gr.
May 9: "Plücker" by Orrin W. Albert Gr.
May 23: "The velocity of a planet" by George H. Cresse Gr.; "The density of a sphere" by Ernest P. Lane Gr.; "Riemann" by Cornelius Gouwens Gr.
June 6: "Infinite determinants" by William G. Simon Gr.
January 16, 1918: "Twisted curves in vector analysis" by Ernest P. Lane Gr.
January 30: "Gauss's reciprocity theorem" by Horace Alson Gr.
February 13: "The limaçon" by Gladys Gibbens Gr.
February 27: "Lagrange" by Charles C. Spooner Gr.
March 13: "Statistical interpretation of entropy" by Edward S. Akeley Gr.
April 17: "The fundamental theorem of algebra" by Ernest Zeisler '19.
May 1: "Some elementary geometric concepts of the complex variable" by Tel C. Kimball Gr.
May 15: "The discovery of Neptune" by Frederick C. Leonard '18.
May 29: "Reflected curves" by Louis Shapotken '18.

THE GRINNELL COLLEGE MATHEMATICS CLUB, Grinnell, Iowa.

This club was founded in March, 1917. At the five meetings during the remainder of the year the programs included the following papers: "The problem of Apollonius" by Professor Raymond B. McClennon; "Planimeters" by Professor William J. Rusk; "Geometry of four dimensions" by Lloyd W. Taylor, Jr., instructor; "Applications of Mathematics to gunnery" by Earl Kilgore '18.

During 1917–18 the club functioned for the first semester only. Linn Smith '19, and Ethel Rivers '17, were respectively president and vice-president. The topics discussed included, "Mathematical fallacies" by Linn Smith '19, and "Squaring the circle" by Fay Breckenridge '18.

THE MATHEMATICAL CLUB, Harvard University, Cambridge, Mass.
[*1918*, 186–7].
October 11, 1916: "Geometrical constructions with special instruments" by Professor Charles L. Bouton.
October 25: "Functions of infinitely many variables" by Dr. William L. Hart, Benjamin Peirce instructor.
November 8: "A classification of closed curves on a surface of finite connectivity by means of a canonical set of closed curves" by Harold C. M. Morse Gr.

[1] Cf. I. Todhunter, *A History of the Mathematical Theory of Probability*, Cambridge and London, 1865, pp. 294 ff.

November 22: "Primary quadratic forms" by Dr. Tracey A. Pierce, instructor.
December 6: "Some intimately related linear equations" by Professor Roland G. D. Richardson of Brown University, Providence, R. I.
January 23, 1917: "Determinants of many dimensions" by Lepine H. Rice Gr.
February 21: "Minkowski's contributions to pure mathematics" by Dr. Edward A. Kircher, Benjamin Peirce instructor.
March 7: "A theory of rectangular matrices" by Professor Clarence L. E. Moore of Massachusetts Institute of Technology.
March 21: "Certain heresies in the teaching of elementary dynamics" by Professor Edward V. Huntington.
April 4: "The fundamental theorem of algebra" by Forrest H. Murray Gr.
May 2: "What is real algebra" by Dr. Henry M. Sheffer, instructor in philosophy.
May 16: "Independent postulates for groups and fields" by Ronald M. Foster '17.

The average attendance at Harvard meetings during 1916–17 was about 26, during 1917–18 about 20.

March 6, 1918: "Calculating machines" by Mr. Modern of the Monroe Calculating Machine Company.
March 20: "Bernhard Riemann" by Professor William F. Osgood.
April 3: "Pencils of Lines" by Dr. Alton L. Miller, instructor.
May 1: "Areal derivatives" by Dr. Gabriel M. Green, instructor.
May 15: "Slide rules" by Professor William R. Ransom of Tufts College, Tufts College, Mass. Professor G. D. Birkhoff was elected faculty adviser, during 1918–19, with power to call a meeting for election of officers.

THE MATHEMATICS CLUB OF THE UNIVERSITY OF KANSAS, Lawrence, Kansas. [*1918*, 35–6].

This club was organized in December, 1911. The following programs supplementing those already given were issued annually in printed form:
October 5, 1914: Election of officers.
October 12: "Mathematics in astronomy" by Professor Ellis B. Stouffer.
October 26: "Addition and subtraction applied to geometry according to the principles of Grassmann" by Edward H. Carus '12, instructor.
November 8: "Who's who in mathematics" by Charles F. Green '14.
November 22: "The fourth dimension" by Laurens E. Whittemore Gr.
January 11, 1915: "The three problems of antiquity" by Cyril A. Nelson Gr.
January 25: "What is mathematics" by Professor Ulysses G. Mitchell.
February 15: "A discussion of the gyroscope" by Wendell M. Latimer '15.
March 8: "Current events in mathematics" by Eva M. Coors '16.
March 22: "The polar planimeter" by Austin Bailey '16.
April 12: "Insurance mathematics" by Professor Charles H. Ashton.
April 26: "Napier and the invention of logarithms" by Ethel W. Mallonee Gr.
May 10: "Quadric surfaces" by Ottilia W. Ducker Gr.
May 24: "A trip to infinity" by Professor John N. Van der Vries.

September 27: Election of officers.
October 11: "Fermat's theorem and allied topics" by Dr. Solomon Lefschetz, instructor.
October 25: "Non-Euclidean geometry" by Jessie Jacobs '15.
November 8: "Line construction" by Ada H. West Gr.
November 22: "Who's who in mathematics in America" by Professor Mitchell; "Mathematical reference books" by Professor Stouffer.
December 13: "Curve tracing" by Paul W. Harnley '15.
January 10, 1916: "Methods of computing errors" by Professor Herbert E. Jordan.
February 14: "Quadratic forms in number theory" by Mabel W. Arnett '15.
February 28: "Elements of orbits of heavenly bodies and Kepler's laws" by Cora J. Shinn '17.
March 3: "Mathematical Fallacies" by James B. Ramsey '16.
March 27: "Some definite integrals" by Arthur W. Larsen, instructor.
April 10: "The origin of the calculus" by Laura J. McKay '16.
April 24: "The mathematics of the calendar" by Leonard L. Steimley, instructor.
May 8: "Finite geometry" by Cyril A. Nelson '14.
May 28: "Review of Memorabilia Mathematica" [Edited by R. E. Moritz] by Florence R. Scheidenberger '15.
September 25: "Methods and customs in German Universities" by Professor Ashton.
October 9: "The mysteries of the fourth dimension" by Margaret Coleman '17.
October 23: "Mathematical games" by Professor Van der Vries.
November 6: "Three famous problems of antiquity" by Hazel E. Parkinson '18.
November 20: "Some simple applications of vector analysis" by Arthur W. Larsen, instructor.
December 11: "The slide rule" by Hobart F. Lutz '19.
January 8, 1917: "Magic squares and cubes" by Cora J. Shinn '18.
January 30: "Japanese and Chinese mathematics" by Frances E. Adams '18.
February 12: "Adding and multiplying machines" by Professor John J. Wheeler.
February 26: "Properties of the number 9" by Marie O. Graff '17.
March 12: "How to draw a straight line" by Earle B. Miller, instructor.
March 26: "Paper folding" by Bernice Boyles '18.
April 9: "Review of De Morgan's Budget of Paradoxes" by Helen R. Garman '18.
April 23: "The planimeter and rectifier" by Lewis M. Hull '17.
May 14: "Mathematical recreations" by Mignonette Uhl '18.
May 25: Picnic.

THE WHITE MATHEMATICS CLUB AT THE UNIVERSITY OF KENTUCKY, Lexington, Ky. [*1918*, 90].

The following list of programs is supplementary to that previously given.
October 11, 1916: "How to draw a straight line" by Professor Paul P. Boyd.

October 18: "Exceptions to the laws of radicals" by Professor Elijah L. Rees.
October 25: "On the contraction of homogeneous spheroids" by Professor Harold H. Downing.
November 1: "Intrinsic equations" by Professor Joseph W. Davis.
November 8: "Theorems relating to the three normals, through a point, to a parabola" by Vernon C. Grove Gr.
November 15: "Elliptic integrals in the problems of the inverse fifth power" by Harry R. Allen Gr.
November 22: "Eclipses" by Homer L. Reid Gr.
December 6: "Graphical construction for a function of a function and for a function given by a pair of parametric equations" [review of W. H. Roever's article in this MONTHLY, 1917] by Clarence W. Harney '17.
December 13: "Linkages" (continued) by Professor Boyd.
December 20: "Trisection of an angle by means of conics" by Professor Rees.
January 6, 1917: "Areas of pedal curves" by Professor Davis.
January 10: "Helmholtz's contraction theory" by Professor Downing.
February 7: "History of Egyptian and Phœnician Mathematics" by Vernon G. Grove Gr.
February 14: "History of Grecian Mathematics" by Harry R. Allen Gr.
February 28: "History of Hindoo and Arabic mathematics" by Homer L. Reid Gr.
March 14: "History of Italian mathematics" by Clarence W. Harney '17.
March 23: "History of English mathematics" by H. L. Reid Gr.
March 28: "History of French mathematics" by V. G. Grove Gr.
April 4: "History of German mathematics" by H. R. Allen Gr.
April 17: "A chart of mathematical history" by Professor Rees.
April 25: "Non-Euclidean geometry" by Myrtle R. Smith '17.
May 1: "The general linear transformation by linkages" by Professor Boyd.
May 9: "Perturbations treated geometrically" by Professor Downing.
May 18: "Pythagoras and the Pythagorean theorem" by Mary Beall '19.
January 16, 1918: "Integrals related to the Lebesgue integrals" by Professor Davis.
January 23: "A discussion of the Rochester plan" [review of W. Betz's article, on "The Teaching of Mathematics in the Junior High School," in *The Mathematics Teacher*, December, 1917] by Professor Davis; "Mathematical requirements in certain high schools" by Miss Mamie Schmidt of the Lexington High School.
February 7: "Review of Townsend and Goodenough's Calculus" by Professor Rees.
February 14: "The cyclo-harmonograph"[1] by Professor Boyd.
February 20: "Arithmetical progression of the *n*th order" by Professor Downing.
February 27: "Introduction to infinitesimal analysis" by Professor Davis.

[1] See R. E. Moritz, (1) "The Cyclo-harmonograph; an instrument for drawing large classes of important higher plane curves," *Scientific American Supplement*, August 5, 1916; (2) On the construction of certain curves given in polar coördinates," in this MONTHLY, May, 1917.

March 6: "A theorem of mechanics proved by vector analysis" by Professor Rees.
March 27: "Point sets" by Professor Downing.
April 3: "Point sets" (continued) by Professor Davis.
April 10: "Point sets" (continued) and "Shortest distance between two points" by Professor Downing.
April 17: "Point sets" (continued) by Professor Davis.
April 24: "Solution of two geometrical problems" by Professor Rees; "Point sets" (continued) by Professor Downing.
May 8: "Photogrammetry" by V. G. Grove Gr.
May 15: "Some applications of vector analysis to kinematics" by H. R. Allen Gr.

THE MATHEMATICS CLUB OF THE UNIVERSITY OF MAINE, Orono, Me. [*1918*, 132].

The following programs are supplementary to those already given:

March 8, 1916: "Simple method of constructing normals to the parabola" [review of S. G. Barton's paper in this MONTHLY, June, 1914] by Maynard F. Jordan '17: "Life of the late Simon Newcomb" by Charles I. Emery '17.
April 5: "Mathematics in the secondary school" by Hoyt D. Foster '16; "Addition formulas for the trigonometric functions" by Marie F. Foster '16; "Fallacies in geometry" by Professor Lowell J. Reed.
April 19: "Methods of teaching high school mathematics" by James A. Hamlin, principal of Old Town High School.
May 31: "Systems of ovals and the ellipse as a special case" by Zella Colvin '16; "Rolling curves" by Raymond D. Douglass '15.
October 11: Social meeting at the home of Professor James N. Hart; there was a short program consisting of competition between several teams in solving problems and mathematical puzzles.
October 25: "Applications of partial derivatives in relation to the study of conics" by Sumner C. Cobb '17; "Life and work of Charles P. Steinmetz"[1] by Clarence H. Drisko '18.
November 8: "On the solutions of linear equations having small determinants" [review of F. R. Moulton's article in this MONTHLY, October, 1913] by Charles I. Emery '17.
November 22: "Curve fitting" by John I. Miner, computer in the Agricultural Experiment Station; "Life of Percival Lowell, astronomer" by Edith De Beck '18.
December 6: "The fourth dimension" by Dr. Norbert Wiener, instructor.
January 17, 1917: "Solution of certain problems prepared in Granville's and Smith's calculus text books with erroneous answers" by Lester C. Swicker '19; "Methods and symbols used in mathematics before the sixteenth century" by Professor Truman L. Hamlin.

[1] Professor of electro-physics at Union College, Schenectady, N. Y.

February 14: "Solution of problems that appeared in the AMERICAN MATHEMATICAL MONTHLY and in *School Science and Mathematics* by Samuel Wiseman '19.

April 18: "Some astronomical topics of current interest" by Professor Hart.

May 16: "A geometrical interpretation of Taylor's formula" by Professor Harley R. Willard.

December 19, 1917: "History of trigonometry" by Edith L. Deering '21; "The nine-point circle" by Edith DeBeck '18.

February 27, 1918: "The eclipse of June, 1918" by Professor Hart; "Comets" by Samuel Guptill '20.

March 27: "Applications of trigonometry to railway curves" by Alpheus C. Lyon, professor of civil engineering; "Ptolemy's theorem and its applications" by Albert J. Bedard '21.

THE MATHEMATICS CLUB OF THE UNIVERSITY OF NORTH CAROLINA, Chapel Hill, N. C. [*1918*, 90–91].

The club members are drawn not only from the department of mathematics but also from the departments of chemistry, physics, and civil and electrical engineering. It has been the aim of the club "to bring teacher and student into closer and more sympathetic contact, to call to the student's attention matters for which opportunity did not present itself in class, to correlate more closely the work of allied departments, to arouse the interest and to foster the development of superior men while lending help and courage to the weaker. The programs have been limited both as to number of papers and as to the time occupied by them, so that problems of interest could be proposed and discussed." These have been conducted by the club's secretary, John W. Lasley, instructor.

The following list of programs supplements that previously given:

November 28, 1916: "The differential coefficient viewed as a singular form" by Professor William Cain; "Mathematical requirements for electrical engineering students" by Elden I. Staples, instructor in electrical engineering.

January 9, 1917: "The logic of mathematics" by Henry H. Williams, professor of philosophy.

February 13: "The inscription of a regular 17-gon in a circle" by Sherman B. Smithey '17; "Some characteristic theorems in the foundations of geometry" by Professor Archibald Henderson.

March 6: "A note on linear equations" by Mr. Lasley; "The teaching of mathematics in the high school" by Lester A. Williams, professor of school administration.

January 7, 1918: "Mathematics historically considered" by Professor Cain. (Lecture open to the public.)

February 6: "A method for finding the complex roots of a cubic equation" by Mr. Lasley.

February 18: "Some aspects of modern geometry" by Professor Henderson (Public lecture).

March 4: "The origin and development of number" by Mr. Lasley (Public lecture).
March 18: "War maps and the use of scales" by Thomas F. Nickerson, associate professor of civil engineering. (Public lecture.)
April 1: "Common sense in mathematics" by Dr. A. Wilson Hobbs, instructor in mathematics. (Public lecture.)
April 22: "Surveying" by Marvin H. Stacy, professor of civil engineering. (Public lecture.)
April 30: "Complex numbers" by Houston Everette '20.

THE MATHEMATICS CLUB OF THE UNIVERSITY OF OREGON, Eugene, Oregon. [*1918*, 134–135].

The remaining programs in 1917–18 were as follows:
February 27: "Solution of the fifth degree equation" by Mary Mottley '19; "The Leibnitz-Newton controversy over the discovery of the calculus" by Glen Langdon '20.
May 15: "Theory of numbers, especially prime numbers, congruences, and Pythagorean numbers" by Olga Soderstrom '18; "Septic curves" by Dr. Roy M. Winger, instructor.

VINCULUM, University of Pennsylvania, Philadelphia, Pa.

This club was organized in May, 1917, "to further interest in mathematics, to provide a seminar for undergraduate students, and to promote sociability among those interested in the subject. Membership is limited to women students, in any department, in any class, who are majoring in mathematics.[1] During the past year there were 20 members and the average attendance at meetings, including visitors, was 25.

Officers 1917–18: President, Edith P. Levinson '18; vice-president, Anna L. Kuhn '20; secretary, Marion George '20; treasurer, Anna Corson '19.

The following is a complete record of meetings (apart from those of a business nature only) from the foundation of the club to the end of the year 1917–18.
November 12, 1917: "Human side of mathematics" by John H. Minnick, assistant professor in education (formerly instructor in mathematics).
December 10: "Early development of arithmetic" by Maurice J. Babb, assistant professor of mathematics.
January 14, 1918: "Application of elementary arithmetic in physics and astronomy" by Professor Edwin S. Crawley.
April 15: "Infinity" by Professor George H. Hallett.

THE MATHEMATICAL CLUB OF SMITH COLLEGE, Northampton, Mass. [*1918*, 91].

October 23, 1916: Social meeting.

[1] At the University of Pennsylvania there were in 1917–18 approximately 900 women—graduate 200, in college courses for teachers 300, in arts 50, and in education 350—all (except graduates) being candidates for bachelor degrees.

November 13: Review of G. A. Miller's *Historical introduction to mathematical literature* by Florence Hatch '17; Review of *A summary of mathematics in the secondary school of tomorrow* by the Mathematics Club of Chicago High School Teachers, by Gertrude Schloo '17.

December 4: Social at the home of Professor Ruth Wood.

January 8, 1917: Review of "On the nature of mathematical reasoning" from Poincaré's *Science and hypothesis* by Muriel Irving '17.

February 12: Review of "Space and Geometry" from Poincaré's *Science and hypothesis* by Lillian Miller '17; Review of "A non-Euclidean world" from J. W. A. Young's *Lectures on fundamental concepts of algebra and geometry* by May E. Owen '17; [title of paper missing in club's records] by Eleanor E. Stearns '17.

March 5: "Lobachevsky's geometry" by Katherine Baxter '17; "Riemann's geometry" by Anna Hebel '18.

March 26: "Bolyai's geometry" by Ruby M. Burt '17;[1] Review of "Experiment and Geometry" from Poincaré's *Science and hypothesis* by Janie B. Bartlett '17.

April 30: "The fourth dimension" by Aline Hueston '17; "The slide rule" by Winifred L. Chase '17; "The integraph" by Professor Pauline Sperry.

May 21: Social meeting at Professor Eleanor P. Cushing's home.

February 11, 1918: Professor Harriet R. Cobb spoke of work at Columbia during two summers; Martha Chadbourne '14 described her graduate work at Harvard and Radcliffe.

March 4: "Pole and polar relations" by Florence B. Putnam '18; "Plane homologies" by Cornelia D. Hopkins '19.

March 25: Professor Ruth Wood spoke on her graduate study at the University of Göttingen; Professor Suzan R. Benedict told of graduate work at the University of Michigan.

April 29: "Brilliant points" by Martha Chadbourne Gr.

May 20: Social meeting.

THE MATHEMATICS CLUB OF VASSAR COLLEGE, Poughkeepsie, N. Y. [*1918*, 136].

February 17, 1916: Organization meeting.

March 15: Election of officers; "Demonstration of Simpson's Rule" by Constance Andrews '16; "Explanation and discussion of the planimeter" by Mildred Allen '16.

April 20: "The equilateral hyperbola" by Frances Atwater '16; "Pascal's hexagon" by Mary McManus '16; "The path and range of projectiles used in modern warfare" by Ruth Core '16; "The construction of shells used in modern warfare" by Helen Moore '16.

May 12: "Some early stages in the evolution of algebra" by Professor Elizabeth B. Cowley.

October 24: Election of Officers; "Professor Mittag-Leffler's last will and testa-

[1] The last three papers were based mainly on Chapter 3 of Poincaré's *Science and hypothesis*.

ment" by Caroline Bacon '17; "Explanation and discussion of the limit" by Roberta Pickering '17.

November 28: "The construction of some curves by linkages" by Helen Moulton '19 and Rachel Franklin '19: "The slide rule" by Katrina Jaggard '17.

December 13: "Lewis Carroll, author and mathematician" by Margaret Finck '17; "Mathematical recreations" by Beatrice Boyden '18.

February 20, 1917: "History of the adding machine" by Maxime Harrison-Berlitz '19; "Explanation and demonstration of the Burroughs adding machine" by Mr. Madison of the Burroughs Machine Co.

February 22: Election of officers.

March 1: "Mathematics in the secondary schools" by Dean Frederick C. Ferry of Williams College.

April 17: "The fourth dimension" by Mary Moher '16; Eleanor Hussey '16, and Mary Appelgate '17.

February 26, 1918: Election of officers for the second semester: President, Martha Braun '18; vice-president, Helen Thompson '19; secretary-treasurer, Louise Stuerm; Professor Cowley (faculty member of executive committee); Christie White '19 (member at large of executive committee); "Planimeters" by Marjorie Wheatley '18 [see *1918*, 136].

April 23: "A mathematical problem of warfare" by Elizabeth B. Conklin '18; "Mathematical fallacies" by Susan Barr '20.

May 7: Picnic.

The number of meetings of the club was smaller in the year 1917–18 than in the previous year because most of its members participated in red-cross work, in farming activities, or in a preparedness course in mechanical drawing. "Those students who wanted to go into government work or to enter offices of electrical companies found this course particularly useful."

THE JUNIOR MATHEMATICAL CLUB, University of Wisconsin, Madison, Wis. [*1918*, 188–9].

October 19, 1916: "American mathematicians" by Professor Edward B. Van Vleck.

November 2: "Graphical representation" by Professor Arnold Dresden.

November 16: "Pappus of Alexandria" by Mary Henry '17; "Models" by Professor Linnaeus W. Dowling.

December 7: "Three breakdowns in mathematics" by Professor Ernest B. Skinner; [title of paper missing from club's records] by Rachel McKannan '17.

January 18, 1917: "Work of Archimedes" by Professor Charles S. Slichter; "Life of Archimedes" by Margaret Chapman '17.

March 1: "Peano's theory of natural numbers" by Frederick Wood, instructor in mathematics.

March 15: [Details of program missing].

March 29: "Squaring the circle" by Professor Walter W. Hart; [title of paper missing] by Stella Andrews '17.

April 19: "Importance of notation" by Dr. Thomas M. Simpson, instructor in mathematics.

May 4: "Geometrical fallacies" by Mary Dillman '17; "Arithmetical fallacies" by Joseph L. Walsh Gr., travelling fellow from Harvard University.

May 18: "Relativity" by Warren Weaver Gr.

April 3, 1918: "Integers" by Professor Skinner.

April 17: "Progress in notation" by Meta Wood '17; "Fractions" by Professor Hart.

May 8: "Lantern slides on mathematicians" by Professor Dresden.

CLUBS REPORTING ON SESSIONS IN 1917–18.

The names of these 37 clubs are arranged chronologically according to the dates of their organization, and pages of this MONTHLY, 1918 where their reports are published are indicated.

The Mathematical and Physical Society of the University of Toronto, Toronto, Ontario.—Founded January, 1882. Membership (men and women) 75; Meetings 9; Average attendance 50. [Pages 229–231.]

The Mathematical Club, Harvard University, Cambridge, Mass.—Founded about 1898. Membership (men only). Meetings 12. Average attendance 20. [Pages 186–187, 449–450.]

The Mathematical Club of Smith College, Northampton, Mass.—Founded October, 1899. Membership (women only) 35. Meetings 11. Average attendance 24. [Pages 91, 455–456.]

Undergraduate Mathematics Club, University of Illinois, Urbana, Ill.—Founded December, 1899. Membership 36. Meetings 8. Average attendance 8–12. [Pages 404–405.]

Pi Mu Epsilon Fraternity, Syracuse University, Syracuse, N. Y.—Founded November, 1903. Membership (men and women) 44. Meetings 9. Average attendance 30. [Pages 271–273.]

Mathematics Club, The Western College for Women, Oxford, Ohio.—Founded 1905. Membership (women only) 28. Meetings 6. [Pages 231–232.]

The Junior Mathematical Club of the University of Chicago, Chicago, Ill.—Founded November, 1905. Membership (men and women) 15–20. Meetings 14. [Pages 34–35, 448–449.]

The Mathematical and Astronomical Club of Swarthmore College, Swarthmore, Pa.—Founded March, 1907. Membership 42. Meetings 16. Average attendance 30. [Page 135.]

Mathematics Club of Mount Holyoke College, South Hadley, Mass.—Founded November, 1907. Membership (women only) 53. Meetings 5. Average attendance 20. [Pages 312–313.]

The White Mathematics Club at the University of Kentucky, Lexington, Ky.—Founded November, 1908. Membership 7. Meetings 22. [Pages 90, 451–453.]

Barnard College Mathematics Club, Columbia University, New York.—Founded 1909. Membership (women only) 40. Meetings 6. Average attendance 25. [Pages 226–227.]

Mathematics Club of Iowa State Teachers College, Cedar Falls, Iowa—Founded December, 1909. Meetings 3. [Pages 311–312.]

The Mathematics Club of Hunter College, New York.—Founded 1910. Membership (women only). Meetings 8. [Pages 187–188.]

Mathematics Club of Columbia University, New York.—Founded November, 1910. Meetings 11. Average attendance 14. [Pages 227–228.]

The Mathematics Club of Albion College, Albion, Mich.—Founded January, 1911. Membership (men and women) 19. Meetings 15. [Pages 354–357.]

The Newtonian Society of the State College of Washington, Pullman, Wash.—Founded November, 1911. Membership (men and women) 14. Meetings 10. Average attendance 10. [Pages 410–411.]

The Mathematics Club of the University of Kansas, Lawrence, Kansas.—Founded December, 1911. Average attendance 19. Meetings 16. [Pages 35–36, 450–451.]

The Junior Mathematical Club, University of Wisconsin, Madison, Wis.—Founded March, 1912. Membership (men and women) 25. Meetings 7. Average attendance 18. [Pages 188–189, 457–458.]

The Mathematics Club of Goucher College, Baltimore, Md.—Founded November, 1913. Membership (women only) 22. Meetings 10. Average attendance 15. [Pages 357–358.]

Denison Mathematics Club, Denison University, Granville, Ohio.) Founded November, 1914. Meetings 11. Average attendance 25. [Pages 403–404.]

Junior Mathematics Club, University of Minnesota, Minneapolis, Minn.—Founded December, 1914. Membership (men and women) 20. Meetings 10. Average attendance 15. [Page 312.]

The Mathematics Club of Brown University, Providence, R. I.—Founded February, 1915. Membership (men and women) 60. Meetings 7. Average attendance 42. [Pages 33–34.]

The Mathematics Club, University of Colorado, Boulder, Colo.—Founded October, 1915. Membership (men and women) 41. Meetings 12. Average attendance 30. [Page 185.]

The Mathematical Club of the University of Nebraska, Lincoln, Neb.—Founded October, 1915. Membership about 60. Monthly meetings 7:30 to 9 P. M. Average attendance 35. [Pages 313–315.]

The Mathematics Club of Northwestern University, Evanston, Ill.—Founded January, 1916. Membership (men and women) 20. Meetings 10. [Pages 132–134, 409.]

The Mathematics Club of the University of Maine, Orono, Me.—Founded February, 1916. Membership (men and women) 18. Meetings 7. Average attendance 8. [Pages 132, 453–454.]

The Mathematics Club of Vassar College, Poughkeepsie, N. Y.—Founded January, 1916. Membership (women only) 37. Meetings 6. [Pages 136, 456–457.]

The Mathematics Club of the University of North Carolina, Chapel Hill, N. C.—

Founded October, 1916. Membership 25. Meetings 10. Average attendance 15–20. [Pages 90–91, 454–455.]

The Mathematics Club of the University of Oregon, Eugene, Oregon.—Founded October, 1916. Membership 30. Meetings 4. Average attendance 20. [Pages 134–135, 455.]

The Pentagram, University of Texas, Austin, Texas.—Founded October, 1916. Membership (men and women) about 30. Meetings 15. [Pages 273–276.]

The Mathematics and Physics Club of the University of Alabama, University, Ala.—Founded November, 1916. Meetings 4. Average attendance 12. [Page 226.]

The Mathematics Club of Greenville College, Greenville, Ill.—Founded September, 1916. Membership (men and women) 25. Meetings 6. Average attendance 20. [Pages 89–90.]

The University of Saskatchewan Mathematical Society, Saskatoon, Sask.—Founded November, 1916. Meetings 6. Average attendance (men and women) 14. [Pages 270–271.]

The Grinnell College Mathematics Club, Grinnell, Iowa.—Founded March, 1917. Membership (men and women). Meetings 6. [Page 449.]

Vinculum, University of Pennsylvania, Philadelphia, Pa.—Founded May, 1917. Membership (women only) 20. Meetings 5. [Page 455.]

Mathematical Club of Rockford College, Rockford, Ill.—Founded October, 1917. Membership (women only) 24. Meetings 9. [Pages 188, 409.]

The Mathematics Club of Connecticut College, New London, Conn.—Founded December, 1917. Membership (women only) 7. Meetings 6. [Page 270.]

Mathematics Club of the University of Montana, Missoula, Montana.—Founded March, 1918. Membership (men and women) 22. Meetings 6. Average attendance 18. [Pages 408–409.]

It may be remarked that more than half of the Clubs were founded within the past four years: that nine were organized in 1916, four in 1917, and one in 1918.

Nearly one quarter of the clubs are for women only: Connecticut, Goucher Hunter, Mt. Holyoke, Pennsylvania, Rockford, Smith, Vassar, and Western.

The number of members in the various clubs varies from less than 10 in such places as Connecticut and Kentucky, to more than 50 at such places as Mt. Holyoke, Brown, Nebraska and Toronto. The number of meetings has varied from 3–5 (for example at Iowa State, Alabama, and Pennsylvania) to 15–22 (for example at Albion, Swarthmore, and Kentucky).

1918—SUMMARY NOTES.

During the year an attempt has been made clearly to indicate the ideals of (1) every American undergraduate mathematical club which held meetings during 1917–18, as well as of (2) the following three clubs which deemed it wise to suspend operation for that year:

The Euclidean Circle of Indiana University, Bloomington, Ind.—Founded September, 1907. [Page 228.]

The Mathematical Club of the Kansas State Agricultural College, Manhattan, Kansas.—Founded September, 1913. [Pages 405–408.]

The Mathematics Club of the University of Oklahoma, Norman, Okla.—Founded October, 1916. [Pages 315–316.]

To this end, in so far as information was procurable each club's organization has been described, and its programs for one or more years have been published.

It is hoped that this material has served, and will serve for some time to come, as a fruitful source of suggestion as to possible methods of club organization and conduct, and as to ideals which may be successfully tried out. A sample constitution (at Oklahoma) has been published, and it has been observed that most clubs favor formal organization with student officers, and a member of the faculty serving in advisory capacity on either the program or executive committee. At such places as Barnard, Saskatchewan, and Toronto, an honorary president is chosen by the students from the faculty, while all other officers are students. Six clubs have student officers only; Iowa State, Kentucky, North Carolina and Syracuse favor faculty control of practically all offices; informality in organizations is characteristic of Brown, Colorado, Goucher and Western. Just what type of control should be maintained by the faculty must often be determined by local conditions.

The facsimile of the certificate of membership at Texas given on page 275, the printed programs of Greenville, Kansas, Kansas State, and Toronto, and the club photographs arranged for at Brown, Greenville, and Indiana, will serve as interesting suggestions.

The address in our October issue, by an experienced teacher who has, for many years, met with great success in his conduct of clubs, sets forth the extreme value of the club and the great importance of considerations involved in the preparation of its programs. During the year more than 650 published programs must have offered helpful suggestions in this connection and made clear to many readers how vital a factor the club may be made in the mathematical, intellectual, and social life of a college.

The Albion program scheme which allots some task to every member at frequent intervals, such plans as those at Goucher, Kansas State, Kentucky, and Wisconsin, for working up portions of the history of mathematics in connection with club programs, and the very general introduction into club programs of biographies of mathematicians, are also noteworthy.

While titles of appropriate program topics are suggestive there are few clubs which do not feel the need of indications of possibilities of the topics, and of exact references to the literature. It was with this idea in view that a beginning was made in meeting this need and the following subjects were treated during the year: 1. The oldest mathematical work extant; 2. Geometrography and other methods of measurement of geometrical constructions; 3. Arithmetical prodigies; 4. Ptolemy's theorem and formulæ of trigonometry; 5. Paper folding; 6. Women as mathematicians and astronomers; 7. The binary scale of notation, a Russian peasant method of multiplication, the game of mim, and Cardan's rings; 8. The

logarithmic spiral; 9. Golden section; 10. A Fibonacci series; 11. Euler integrals and Euler's spiral—sometimes called Fresnel integrals, and the clothoïde, or Cornu's spiral; 12. Geometry of four dimensions; 13. Constructions with a double-edged ruler; 14. The cattle problem of Archimedes.

With one exception (because calculus is involved) all of these topics are suitable for programs of every one of the clubs mentioned above. In most cases far more was suggested than was regarded as possible for consideration in a single evening. The bibliographies were purposely made pretty full, partly because the synopses were contributions to mathematical history, and partly that the resources of each library might be laid under contribution when possible. It was not intended that anything of real importance and interest should have been overlooked; nevertheless it was not noted till too late that (1) Campanus, in the thirteenth century, proved the irrationality of golden section and that his argument (by mathematical induction) was reproduced by Genocchi and Cantor;[1] that (2) discussion connected with Fibonacci's series occurs in the writings of Daniel Bernorilli as early as 1728 and of Euler by 1726 (facts to which Mr. G. Eneström of Stockholm has kindly drawn my attention); and that (3) Euler's spiral has played an interesting and important rôle in connection with the history of railroad curves. The literature of this subject has been surveyed by the editor and it is possible that at a later date the results of his inquiry may be thrown into the form of a club topic.

* * *

The following passage of a personal letter to the editor, from a distinguished professor in a Scottish University, will be of general interest.

"But what I want most to write about today is to thank you for, and to offer you a word of appreciation of, your bulletin of the 'Undergraduate Clubs.' We have nothing like this over here, and the whole thing strikes me as admirable. It seems to me that it might with immense advantage be extended (as perhaps it already is on your side) to other sciences as well as to mathematics. Our better students either drop their work altogether on graduation, or either insist on attempting, or perhaps are tempted (for instance by the Carnegie Scholarships) to attempt, 'original investigation' before they are fit for it. There is no temptation and little opportunity to prolong their own studies, to engage in wider reading or to make acquaintance with the historical aspect of their science. Often a comparatively raw youth or girl comes to me and wants to 'do original research.' I say 'Why, you have read nothing but a text-book or two; you have read nothing worth speaking of. Why not read for a year or two; make yourself master of what has been done in this field or that, and widen your horizon. Epitomize the literature of some theme, or of some historical period.' But the reply is always the same. *'I want to do "original research"*; and the Carnegie

[1] *Annali di scienze matematiche e fisiche* (Tortolini), tomo 6, pp. 307–308; Cantor's *Vorlesungen über Geschichte der Mathematik*, Band 2, 2. Auflage, 1900, p. 105. See also this MONTHLY, 1918, page 197.

Trust will give me a good scholarship for doing so, and for nothing else in the world.' Now your plan seems to me to precisely meet the case. You encourage your young people to do *work*—not necessarily 'original work'—which is more than any reasonable man can expect of them; but at least work which involves just so much originality or at least independence as can fairly be expected. And it is work by which they learn something; while heaps of so-called original work, as I see it done, teaches nothing, for it is too often confined to some tiny problem, and only means watching the spot of a galvanometer, or making endless and all but identical titrations or measurements."

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